ON A QCD-BASED PION DISTRIBUTION AMPLITUDE VS. RECENT EXPERIMENTAL DATA

A.P. BAKULEV[1, a], S.V. MIKHAILOV[1], N.G. STEFANIS[2]
[1] *Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia*
[2] *Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

Using QCD sum rules with nonlocal condensates the twist-2 pion distribution amplitude is determined by means of its moments and their confidence intervals, including also radiative corrections. An admissible set of pion distribution amplitudes is constructed in the a_2, a_4 plane of the Gegenbauer polynomial expansion coefficients. The determined a_2, a_4 region strongly overlaps with that extracted from the CLEO data by Schmedding and Yakovlev. Comparisons are given with results from Fermilab experiment E791 and recent lattice calculations.

1 Pion Distribution Amplitude

The main object of this talk is the pion distribution amplitude (DA), defined by

$$\langle 0 \mid \bar{d}(z)\gamma_\mu\gamma_5 E(z,0)u(0) \mid \pi(P)\rangle\Big|_{z^2=0} = i f_\pi P_\mu \int_0^1 dx\ e^{ix(zP)}\ \varphi_\pi(x,\mu^2)\ , \qquad (1)$$

where gauge invariance is ensured by the Fock-Schwinger string $E(z,0) = \mathcal{P}\exp\left[ig\int_0^z A_\mu(t)dt^\mu\right]$.

The pion DA has the following important properties: (1) it is multiplicatively renormalizable, (2) it has isospin symmetry: $\varphi_\pi(1-x,\mu^2) = \varphi_\pi(x,\mu^2)$, and (3) its normalization is conserved: $\int_0^1 dx\ \varphi_\pi(x,\mu^2) = 1$.

Pion DAs naturally appear in the perturbative QCD (pQCD) description of any *hard exclusive* process with pions. For example, the form factor of $\gamma^*\gamma^* \to \pi^0$ decay with $-q_{1,2}^2 \sim Q^2 \geq 1\ \mathrm{GeV}^2$ is factorized in pQCD according to

$$F_{\gamma^*\gamma^*\to\pi^0}(q_1^2,q_2^2) = C(q_1^2,q_2^2;\mu^2;x) \otimes \varphi_\pi(x;\mu^2) + O\left(Q^{-4}\right) + \dots \qquad (2)$$

[a] talk presented by this author

For $\varphi_\pi(x)$ we use an expansion in terms of eigenfunctions of the 1-loop evolution kernel, $x\bar{x}C_n^{3/2}(2x-1)$,

$$\varphi_\pi(x;\mu^2) = \varphi_\pi^{\rm as}(x)\Big[1 + a_2(\mu^2)C_2^{3/2}(\xi) + a_4(\mu^2)C_4^{3/2}(\xi) + \ldots\Big] \qquad (3)$$

with $\varphi_\pi^{\rm as}(x) \equiv 6x\bar{x}$ being the asymptotic pion DA and $\xi \equiv 2x-1$. In this expansion all scale-dependence is accumulated in the coefficients $\left\{a_2(\mu^2), a_4(\mu^2), \ldots\right\}$. Note that the evolution of the pion DA at the 2-loop level is available from [1,2].

How one can obtain the $\varphi_\pi(x,\mu^2)$? It is possible to extract it from:

- experimental data: (i) see the recent papers of the CLEO Collaboration [3] and the analysis of Schmedding and Yakovlev of these data [4], and (ii) using the data of the E791 Collaboration [5]
- QCD Sum Rules with Non-Local Condensates (NLC) see [6,7,8]
- transverse lattice simulations [9,10]
- instanton-induced models [11,12].

In this talk we consider all these sources separately, but the main focus is on the first 2 items.

2 Revision of the NLC QCD SR Results

We re-analyze our NLC SRs with the modification of one of the antiquark-gluon-quark NLC contributions to obtain revised values of the moments $\langle\xi^N\rangle_\pi = \int_0^1 \varphi_\pi(x;\mu^2)\,(2x-1)^N\,dx$ ($N = 2,\ldots,10$), new estimates of error-bars, and a new estimate of $\langle x^{-1}\rangle_{SR} = \int_0^1 \varphi_\pi(x;\mu^2)x^{-1}dx$, where a SR is used that is constructed directly for this quantity.

Our model of NLCs, illustrated by the scalar NLC $\langle \bar{q}(0)q(x)\rangle = \langle \bar{q}(0)q(0)\rangle \exp\left(-|x^2|\lambda_q^2/8\right)$, uses only a single parameter λ_q^2, which is related to the vacuum (fields) correlation length $1/\lambda_q = \Lambda$ the latter being of the order of the hadron size, as estimated in non-pQCD approaches:

$$\lambda_q^2 = \left\{\begin{array}{ll} 0.4 \pm 0.1~\text{GeV}^2 & \left[\text{ QCD SRs, 1983 [13] }\right] \\ 0.5 \pm 0.05~\text{GeV}^2 & \left[\text{ QCD SRs, 1991 [14] }\right] \\ \approx 0.5~\text{GeV}^2 & \left[\text{ Lattice, 1998-99 [15,16] }\right] \end{array}\right. \qquad (4)$$

From the lhs of Fig. 1, one sees that the quality of the stability in the NLC QCD SRs for $\lambda_q^2 = 0.4$ GeV^2 is quite high (solid line stands for the best threshold $s_0 = 2.2$ GeV^2, dashed lines – for 10%-variations of this parameter). The obtained moments are shown on the lhs of Fig. 2.

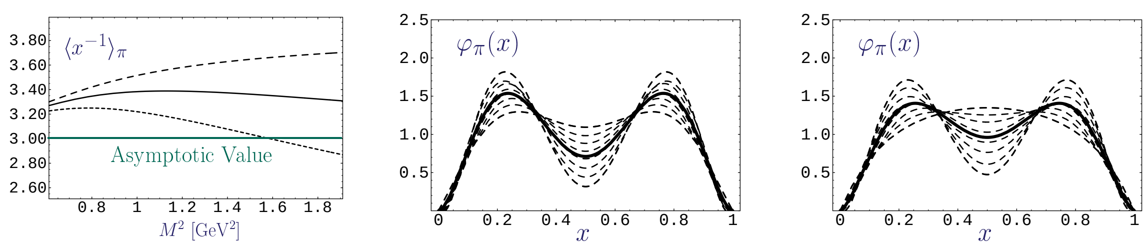

Figure 1: Left side: stability curves for $\langle x^{-1}\rangle_\pi$ against the Borel parameter M^2. Central part: Bunch of admissible DAs corresponding to $\lambda_q^2 = 0.4$ GeV^2 with best-fit parameters $a_2 = +0.188$, $a_4 = -0.130$. Right side: Same as central part, but for $\lambda_q^2 = 0.5$ GeV^2 with best-fit parameters $a_2 = +0.126$, $a_4 = -0.091$.

We reconstruct the pion DA from these five moments, using models (3) at $\mu^2 = 1$ GeV^2 with two non-zero Gegenbauer coefficients. The best-fit DAs obtained this way (with $\chi^2 \approx 10^{-3}$) are shown in Fig. 1 as thick solid lines. The corresponding error bars to the DAs, allowed by the moment SRs, are also shown. The bunches of these broken lines represent the self-consistent DAs in the sense that the value of the associated inverse moment, $\langle x^{-1}\rangle_\pi = 3.17(8)$ ($\langle x^{-1}\rangle_\pi = 3.13(8)$, for $\lambda_q^2 = 0.5$ GeV^2), is in good agreement with the value determined from the special SR: $\langle x^{-1}\rangle_\pi^{\rm SR} = 3.33(32)$ ($\langle x^{-1}\rangle_\pi^{\rm SR} = 3.19(32)$).

3 Comparing with CLEO results in SY approach

Schmedding and Yakovlev [4] have provided a useful analysis of the CLEO data on the $\gamma^*(q)\gamma \to \pi^0$ form factor [3], using light-cone QCD SRs and taking into account NLO and twist-4 corrections. They estimated the first two Gegenbauer coefficients of the pion DA and obtained

$$a_2 = 0.19 \pm 0.04(\text{stat}) \pm 0.09(\text{sys})\ , \quad a_4 = -0.14 \pm 0.03(\text{stat}) \mp 0.09(\text{sys}) \tag{5}$$

with results displayed in Fig. 2 in the form of confidence regions in terms of a_2 and a_4.

We evolve our allowed sets to the CLEO scale $\mu^2 = (2.4\ \text{GeV})^2$ and insert them directly into the SY plot to obtain results in the a_2, a_4 plane, shown in Fig. 2. Inspection of these plots

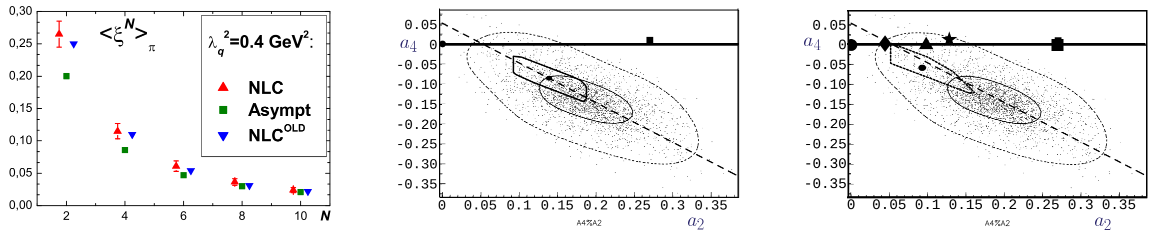

Figure 2: Left: $\langle \xi^N \rangle$ against N. Central part: Confidence region of pion DAs for $\lambda_q^2 = 0.4$ GeV^2. Right side: the same as before with specific explanations given in the text, but with $\lambda_q^2 = 0.5$ GeV^2.

reveals that bunch-1 $\left[\lambda_q^2 = 0.4\ \text{GeV}^2\right]$ and bunch-2 $\left[\lambda_q^2 = 0.5\ \text{GeV}^2\right]$ are intersecting with the SY 1σ-region and are thus quite *compatible* with CLEO data.

Inspecting the SY plot, one natural question arises: why the confidence regions are stretched along the diagonal $a_2 + a_4 =$ const ? To answer this question, it is instructive to analyze the CLEO data, employing pure pQCD:

$$\frac{3Q^2}{4\pi} F_{\gamma\gamma^*\pi} \approx Q^2 \cdot C \otimes \varphi_\pi \ = \ \langle x^{-1} \rangle_\pi + 3\alpha_s \left(\Delta_0 + \Delta_2 + \Delta_4\right) + \ldots \tag{6}$$

We see, that, up to radiative corrections, CLEO in fact measured the inverse moment of the pion DA, $\langle x^{-1} \rangle_\pi$, which is simply connected to the diagonal combination of the a_2 and a_4 coefficients:

$$\langle x^{-1} \rangle_\pi = 3\left(1 + a_2 + a_4\right) \tag{7}$$

The SY analysis gives: $a_2 + a_4 = 0.05 \pm 0.07$, whereas the special NLC QCD SRs yields for this moment $\frac{1}{3}\langle x^{-1} \rangle_\pi^{\text{SR}} - 1 = 0.10 \pm 0.10$ (at $\mu = 1$ GeV), and our bunch-1 of the allowed pion DAs produces $a_2 + a_4 = 0.056 \pm 0.03$ (at $\mu = 2.4$ GeV), in excellent agreement with S&Y result.

It is useful to have a look on the numerical values of different terms in (6)

$$\begin{aligned} \frac{3Q^2}{4\pi} F_{\gamma\gamma^*\pi} \ &\approx \ 3\left[1 + (a_2 + a_4) + \alpha_s\Delta_0 + \alpha_s\left(\Delta_2 + \Delta_4\right) + \ldots\right] \\ &= \ 3\left[1 + \quad 0.05 \quad - \ 0.17 \ - \quad 0.014 \quad + \quad \ldots\right] . \end{aligned} \tag{8}$$

The CLEO data gives numerically $\left(3Q^2/4\pi\right) F_{\gamma\gamma^*\pi} \approx 2.45$. We see that in the LO pQCD analysis (i.e., when all $\alpha_s\Delta_N = 0$), one arrives at the estimate: $\langle x^{-1} \rangle_\pi = 2.45$. From this, one might conclude that $\varphi_\pi(x)$ should be *narrower* than the asymptotic one [17]. Taking into account only the main part of the NLO correction ($\alpha_s\Delta_0 = -0.17$), we get the estimate, $\langle x^{-1} \rangle_\pi = 2.96$, and conclude that $\varphi_\pi(x)$ could have the same width as the asymptotic DA. But the full NLO (plus twist-4 contribution) light-cone QCD SR [4] provides instead, $\langle x^{-1} \rangle_\pi \approx 3.15$, indicating that $\varphi_\pi(x)$ *should be broader than the asymptotic DA* [18], just as it appears in our bunches in Fig. 1.

The E791 collaboration has measured dijet production in diffractive πA interactions [5]. Such an experiment has been suggested in 1993 in [19] as a means of measuring *directly* the squared pion

DA at large transverse momentum transfers. We obtain a good fit of the E791 data using our model (symbol $\star$ on the rhs of Fig. 2) with $a_2^{\rm fit}=0.12$ and $a_4^{\rm fit}=0.01$ at the scale ~ 8 GeV^2. The resulting value of the "diagonal" at the CLEO scale appears to be too large: $a_2^{\rm fit}+a_4^{\rm fit}\simeq 0.14$. In our view, the interpretation of this experiment is still questionable. Moreover, it seems that errors are too large and should be estimated more carefully.

There are two recent papers involving transverse lattice simulations. Dalley [9] produced (see symbol ▲ on the rhs of Fig. 2)

$$\varphi_\pi^{\rm lat}(x;\mu^2\simeq 1~{\rm GeV}^2)=6x\bar{x}\Big[1+0.133C_2^{3/2}(2x-1)\Big]\,, \tag{9}$$

and, on the other hand, Burkardt and Seal [10] arrived – using the same approach – at a different DA (denoted by • in Fig. 2) very close to the asymptotic pion DA. Note that this large difference seems to indicate that the errors of this method are still large and should be estimated more precisely.

The existing predictions from instanton-induced models [11,12] are too close to $\varphi_\pi^{\rm as}$, except for the new model by Praszałowicz and Rostworowski [20], which is just outside the confidence region of the φ_2-bunch and on the boundary of the 95%-region of SY (symbol ♦ on the rhs of Fig. 2).

Acknowledgments

This work was supported in part by the Russian Foundation for Fundamental Research (contract 00-02-16696), INTAS-CALL 2000 N 587, the Heisenberg–Landau Program (grants 2000-15 and 2001-11), and the COSY Forschungsprojekt Jülich/Bochum.

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